Directions: Find each of the following limits by hand, simplify your answers. Use appropriate notation.

21. $\lim_{x\to -1}\frac{2x^2-x-3}{x+1}$ $= \frac{(2x-3)(x+1)}{x+1}$

$\lim_{x\to -1}\frac{2x^2-x-3}{x+1} = -5$

22. $\lim_{x\to -1}\frac{x^3+1}{x+1}$ $= \frac{(x+1)(x^2+1-x)}{(x+1)}$

$1+1+1=3$

$\lim_{x\to -1}\frac{x^3+1}{x+1} = 3$

23. $\lim_{x\to 5}\frac{5-x}{x^2-25}$ $= \frac{-(x-5)}{(x-5)(x+5)}$

$\frac{-1}{5+5} = -\frac{1}{10}$

$\lim_{x\to 5}\frac{5-x}{x^2-25} = -\frac{1}{10}$

24. $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$ $\cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \frac{(x-3)}{(x-3)(\sqrt{x+1}+2)}$

$\frac{1}{\sqrt{3+1}+2} = \frac{1}{4}$

$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3} = \frac{1}{4}$

25. $\lim_{x\to 0}\frac{\sin x}{x}$ $= 1$

26. $\lim_{x\to 0}\frac{\sin 5x}{x}$ $\cdot \frac{5}{5} = \frac{5\sin 5x}{5x}$

$\lim_{x\to 0}\frac{\sin 5x}{x} = 5$

Directions: Complete the problems below.

27. Determine the value for a and b to ensure that f(x) is continuous.

$$f(x) = \begin{cases} x-1, if\ x \le -1 \\ ax+b, -1 < x < 1 \\ 2x+1, x \ge 1 \end{cases}$$

$\frac{1}{2} - a = -2$

$-a = -\frac{5}{2}$

$a = \frac{5}{2}$

$b - a = -2$

$a + b = 3$

$a = 3 - b$

$b - 3 + b = -2$

$2b = 1$

$b = \frac{1}{2}$

28. Given $\lim_{x\to 3} f(x) = 7$ and $\lim_{x\to 3} g(x) = 3$, find:

A. $\lim_{x\to 3}(f(x)+g(x))$ $= 10$

B. $\lim_{x\to 3}\left(\frac{f(x)}{g(x)}\right)$ $= \frac{7}{3}$

C. $\lim_{x\to 3}(f(x)g(x))$ $= 21$

Directions: Determine whether or not the Intermediate Value Thm applies in each situation. If so, find c.

29. $f(x) = x^2 - 4$ $\quad f(c) = 221\ on\ [4,20]$

$221 = x^2 - 4$

$225 = x^2$

$x = \pm 15$

f(x) is continuous & $f(4) = 12$ & $f(20) = 396$ ∴ a c exists so $f(c) = 221$ by IVT

$c = 15$

30. $f(x) = \frac{x-1}{x^2-2x+1}$ $\quad f(c) = -1\ on\ (-1,1)$

$(x-1)(x-1)$

f(x) is not continuous from [-1,1] so IVT doesn't apply

Directions: Use the definition of the derivative to find $f'(x)$ or $f'(t)$. Show correct limit symbolism.

31. $f(x) = x^2 - 1$

$\lim_{h\to 0}\frac{(x+h)^2-1-x^2-1}{h} = \frac{\cancel{x^2}+2xh+h^2\cancel{-1-x^2+1}}{\cancel{h}} = 2x+h$

$f'(x) = 2x$

32. $f(t) = t^3 - 12t$

$\lim_{h\to 0}\frac{(t+h)^3-12(t+h)-t^3+12t}{h} =$

$\lim_{h\to 0}\frac{\cancel{t^3}+3t^2h+3h^2t+h^3-12t-12h-\cancel{t^3}+12t}{h}$

$\lim_{h\to 0} 3t^2+3h+h^2-12$

$= 3t^2 - 12$

$f'(t) \quad 3t^2-12$

Directions: Use the alternate form of the limit definition of the derivative to find the indicated derivative.

33. $f(x) = x^2 - 1, f'(2)$

$\lim_{x\to 2}\frac{x^2-1-3}{x-2} = \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{(x-2)}$

$f'(2) = 4$

34. $f(x) = \frac{1}{x}, f'(3)$

$\lim_{x\to 3}\frac{\frac{1}{x}-\frac{1}{3}}{x-3} = \frac{\frac{3-x}{3x}}{x-3} =$

$\lim_{x\to 3}\frac{-(x-3)}{3x(x-3)} = \frac{-1}{3x}$

$f'(3) = -\frac{1}{9}$